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Steady MHD Free Convection and Mass Transfer Flow with Hall and Ion-Slip Current in a Rotating System

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Abstract: The steady MHD free convection and mass transfer flow of a viscous, incompressible, electrically conducting and partially ionized fluid past an impulsively started semi- infinite vertical porous plate with thermal diffusion, Hall and ion-slip currents and large suction in a rotating system under the influence of a transversely applied magnetic field is studied. The whole system is assumed to be in a state of rigid body rotation. Similarity variables are introduced to transfer the governing equations and the short circuit situation is considered to simplify the equations. Under the assumption of large suction and using perturbation technique the simplified equations have been obtained and are then solved in straightforward manner. The results are shown graphically as well as tabular for different values of the non-dimensional parameters.

Key words: Rotating system, Thermal diffusion, Hall current, Ion-slip current and Large suction

Introduction

Many researchers have studied magnetohydrodynamics free convection flow of an electrically conducting fluid along a heated semi-infinite vertical flat plate in the presence of a strong magnetic field. In most of the studies, Hall and ion-slip terms were ignored in applying Ohm's law, as it has no marked effect for small and moderate values of the magnetic field. However, the current trend for application of magnetohydrodynamics is towards a strong magnetic field, so that the effect of electromagnetic force is noticeable Cramer and Pai [4]. Under these conditions Hall and ion-slip currents are important and they have a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term. The problem of MHD free convection flow with Hall and ion-slip currents has many important engineering applications, e.g. in power generators, Hall accelerators and flows in channels and ducts. Bo-Eldahab and El Aziz [1] studied the effect of Hall and ion-slip currents with internal heat.

Ram and Takhar [6] studied the MHD free convection flow past an infinite vertical plate with Hall and ion-slip currents when the fluid and the plate are in a state of rigid rotation. Hossain [5] has considered the effects of Joule heating for MHD forced and free convection flow. Alam and Sattar [2] studied the steady two-dimensional MHD free convective and mass transfer flow with thermal diffusion and large suction past an infinite vertical porous plate in a rotating system. An important type of rotating boundary

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layer flow is the flow over rotating blades, occurring in turbines, helicopters and propellers. In view of the above investigations, in the present work, we have studied the MHD free convection and mass transfer flow with thermal diffusion, Hall current and ion-slip current and large suction of a viscous incompressible, electrically conducting and partially ionized fluid past an impulsively started infinite vertical plate. The whole system is assumed to be in a state of rigid body rotation. The governing non-linear complete partial differential equations are transformed to a set of non-linear coupled ordinary differential equations by introducing similarity variables.

Considering the case of short circuit condition the equations are simplified. Under the condition of large suction, following Singh and Dikshit [7] and Bestman [3], the dependent variables have been expanded in terms of small perturbation quantity and considering the order of this quantity the set of ordinary differential equations are obtained. The boundary conditions have also been transformed accordingly and the set of ordinary differential equations, whose coefficients are non-dimensional parameters, have been solved in straightforward manner. The effects of various parameters have been calculated and are shown in graphically as well as tabular form.

Governing Equations

The steady MHD free convection flow of a viscous, incompressible and electrically conducting partially ionized fluid past an impulsively started infinite vertical porous plate with thermal diffusion, Hall current, ion slip current and large suction in a rotating system under the influence of a transversely applied magnetic field is considered in the present flow model. Let the *x* and *y*-axis be along and normal to the plate respectively. Let *u* and *v* are the velocity components along *x* and *y* directions respectively. Initially the plate and the fluid are at rest and the temperature of the fluid and the plate are also same. The plate temperature and the fluid concentration are instantly raised from T_∞ and C_∞ to T_w and $C_w(x)$ respectively, where T_∞ and C_∞ are the temperature and concentration of the uniform flow constant. The induced magnetic field is assumed to be negligible so that $B=(0,B_0,0)$ where B_0 is the constant transversely applied magnetic field which is acting along the direction of y .

If J=$\left(J_x, J_y, J_z\right)$ is the current density, the equation of conservation of electric charge ∇.J= 0 gives $J_y =$constant. Since the plate is electrically non-conducting, this constant is zero and hence, $J_y = 0$ everywhere with in the flow. We further consider that the fluid and the plate are in a state of solid body rotation with a constant angular velocity **Ω** about *y*-axis, which is taken to be perpendicular to the plate. The flow configuration in a rotating system is shown in figure 1. Within the framework of such assumptions and under Boussinesq's approximation, the equations relevant to the problem can be put in the following form:

The continuity equation $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$ (1)

The momentum equations

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-2w\Omega=\nu\frac{\partial^2 u}{\partial y^2}+g_0\beta^{'}(T-T_\infty)$$
$$+g_0\beta^{*}(C-C_\infty)-B_0\frac{J_z}{\rho} \qquad (2)$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+2u\Omega=\nu\frac{\partial^2 w}{\partial y^2}+B_0\frac{J_x}{\rho} \qquad (3)$$

The energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y^2} \qquad (4)$$

The concentration equation

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_M\frac{\partial^2 C}{\partial y^2}+D_T\frac{\partial^2 T}{\partial y^2} \qquad (5)$$

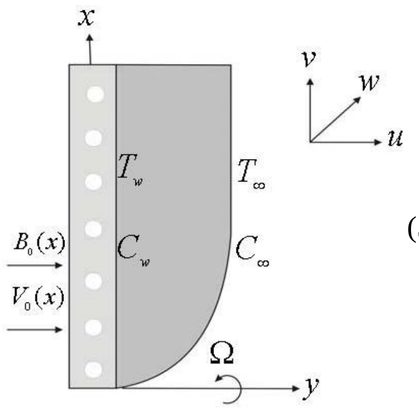

Fig. 1: *Physical configuration and coordinate system*

where Ω is the angular velocity, g_0 *i*s the acceleration due to gravity, β' is the co-efficient of volume expansion, $\beta^\bullet$ is the co efficient of expansion with concentration, ρ is the density, T is the temperature of the flow field, T_∞ is the temperature of the fluid at infinity, C is the species concentration, C_∞ is the species concentration at infinity, D_m is the molecular diffusivity, D_T is the thermal diffusivity,

$J_z=\alpha[E_z+B_0u]\sigma-\beta[E_x-B_0w]\sigma$ and $J_x=\alpha[E_x-B_0w]\sigma+\beta[E_z+B_0u]\sigma$ in which $\alpha=(1+\beta_i\beta_e)/\left((1+\beta_i\beta_e)^2+\beta_e^2\right)$, $\beta=\beta_e/\left((1+\beta_i\beta_e)^2+\beta_e^2\right)$, β_i is the ion-slip parameter and β_e is the Hall parameter.

The boundary conditions for the problem are:

$$\left.\begin{array}{ll} u=U_0,\ \ v=v_0(x),\ \ w=0,\ \ T=T_w,\ \ C=C_w(x) & \text{at}\ \ y=0 \\ u=0,\qquad\qquad\qquad w=0,\ \ T=T_\infty,\ \ C=C_\infty & \text{at}\ \ y\to\infty \end{array}\right\} \qquad (6)$$

where U_0 is the uniform velocity and $v_0(x)$ is the suction velocity at the plate and $C_w(x)$ is the variable concentration at the plate.

Mathematical Formulations

To attain similarity solutions we have introduced the following similarity variables:

$$\eta=y\sqrt{\frac{U_0}{2\nu x}}\ ,\ f'(\eta)=\frac{u}{U_0},\ g(\eta)=\frac{w}{U_0},\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{x(C_0-C_\infty)} \qquad (7)$$

Now for reasons of similarity the plate concentration $C_w\,(x)$ is taken to be

$$C_w(x) = C_\infty + (C_0 - C_\infty)\bar{x} \tag{8}$$

where $\bar{x} = \frac{xU_0}{\nu}$ and C_0 is considered to be the mean concentration.

Now in terms of (8) the equation (1) can be integrated to

$$v = \sqrt{\frac{\nu U_0}{2x}}\,(\eta f' - f) \tag{9}$$

Introducing equation (7) into the equations (2)-(5), we obtain the similarity equations

$$f''' + ff'' + 4Rg - M_1 f' - M_2 g + G_r\theta + G_m\phi - N' + N_2' = 0 \tag{10}$$

$$g'' + g'f - 4Rf' + f'M_2 - M_1 g + N_1 + N_2 = 0 \tag{11}$$

$$\theta'' + P_r f\theta' = 0 \tag{12}$$

$$\phi'' - 2S_c f'\phi + S_c\phi' + S_0 S_c\theta'' = 0 \tag{13}$$

where

$G_r = \frac{g_0\beta' 2x(T_w - T_\infty)}{U_0^2}$ (Local Grashof number)

$G_m = \frac{g_0\beta^{\bullet} 2x^2(C_0 - C_\infty)}{\nu U_0}$ (Local modified Grashof number)

$S_0 = \frac{D_T(T_w - T_\infty)}{xU_0(C_w - C_\infty)}$ (Local Soret number),

$P_r = \frac{\rho\nu C_p}{k}$ (Prandtl number)

$S_c = \frac{\nu}{D_M}$ (Schmidt number)

$M = \frac{2xB_0^2\sigma}{\rho U_0^2}$ (Local magnetic parameter),

$R = \frac{x\Omega}{U_0}$ (Rotational parameter)

$N_1' = \frac{2xB_0\alpha\sigma E_z}{U_0^2\rho}$ $N_2' = \frac{2xB_0\sigma\beta E_x}{U_0^2\rho}$, $N_1 = \frac{2xB_0\alpha\sigma E_x}{U_0^2\rho}$, $N_2 = \frac{2xB_0\beta\sigma E_z}{U_0^2\rho}$

$M_1 = \alpha M,\ M_2 = \beta M$

We now consider further the case of short circuit problem in which the applied electric field $E = 0$. Now for this we have $N_1 = 0, N_2 = 0, N_1' = 0,$ and $N_2' = 0.$ Then the above equations (10) and (11) reduces to the following form and the other two equations remain same.

$$f''' + ff'' + 4Rg - M_1 f' - M_2 g + G_r \theta + G_m \phi = 0 \quad (14)$$

$$g'' + g'f - 4Rf' + f'M_2 - M_1 g = 0 \quad (15)$$

The corresponding boundary conditions are

$$\left. \begin{array}{l} f = f_w,\ f' = 1,\ g = 0,\ \theta = 1,\ \phi = 1 \text{ at } \eta = 0 \\ f' = 0,\ g = 0,\ \theta = 0,\ \phi = 0 \text{ as } \eta \to 0 \end{array} \right\} \quad (16)$$

where $f_w = -v_0(x)\sqrt{\frac{2x}{\nu U_0}}$ is the transpiration parameter and primes denote derivative with respect to η. Here $f_w > 0$ indicates the suction and $f_w < 0$ the injection. The solution of the equations (12)-(13) & (14)-15) subject to the boundary conditions (16) are now sought and are presented in the following section.

Solutions

To solve the equations the following transformations are made

$$\zeta = \eta f_w, f(\eta) = f_w F(\zeta), g(\eta) = f_w^2 G(\zeta), \theta(\eta) = f_w^2 H(\zeta), \phi(\eta) = f_w^2 P(\bar{\zeta}) \quad (17)$$

Substituting (17) in equations (12) –(13) & (14) –(15) we have

$$F''' + FF'' + \varepsilon\left\{-M_1 F' + \left(4R - M_2\right)G + G_r H + G_m P\right\} = 0 \quad (18)$$

$$G''' + G'F + \varepsilon\left\{F'\left(M_2 - 4R\right) - M_1 G\right\} = 0 \quad (19)$$

$$H'' + P_r F H' = 0 \quad (20)$$

$$P'' - 2S_c F'P + S_c FP' + S_0 S_c H'' = 0 \quad (21)$$

where $\varepsilon = \frac{1}{f_w^2}$

The transformed boundary conditions are

$$\left. \begin{array}{l} F = 1,\ F' = \varepsilon,\ G = 0,\ H = \varepsilon,\ P = \varepsilon \text{ at } \zeta = 0 \\ F' = 0,\ G = 0,\ H = 0,\ P = 0 \text{ as } \zeta \to \infty \end{array} \right\} \quad (22)$$

Now for large suction $f_w >> 1$, so that ε is very small, therefore, following Singh and Dikshit [7] and Bestman [3], F, G, H and P can be expanded in terms of small perturbation quantity ε as

$$F(\zeta) = 1 + \varepsilon F_1(\zeta) + \varepsilon^2 F_2(\zeta) + \varepsilon^3 F_3(\zeta) + \cdots\cdots \quad (23)$$

$$G(\zeta) = \varepsilon G_1(\zeta) + \varepsilon^2 G_2(\zeta) + \varepsilon^3 G_3(\zeta) + \cdots\cdots \quad (24)$$

$$H(\zeta) = \varepsilon H_1(\zeta) + \varepsilon^2 H_2(\zeta) + \varepsilon^3 H_3(\zeta) + \cdots\cdots \quad (25)$$

$$P(\zeta) = \varepsilon P_1(\zeta) + \varepsilon^2 P_2(\zeta) + \varepsilon^3 P_3(\zeta) + \cdots\cdots \quad (26)$$

Then substituting $F(\zeta)$, $G(\zeta)$, $H(\zeta)$, and $P(\zeta)$, from (23)–(26) in the equations (18)–(21), we have the following set of ordinary differential equations and the boundary conditions for $F_i(\zeta)$, $G_i(\zeta)$, $H_i(\zeta)$ and $P_i(\zeta)$ $(i = 1, 2, 3, ...)$;

For the first order; *0 (ε)*:

$$F_1''' + F_1'' = 0 \tag{27}$$

$$G_1'' + G_1' = 0 \tag{28}$$

$$H_1'' + P_r H_1' = 0 \tag{29}$$

$$P_1'' + S_c P_1' + S_0 S_c H_1'' = 0 \tag{30}$$

$$\left.\begin{array}{l} F_1 = 0,\ F_1' = 1,\ G_1 = 0,\ H_1 = 1,\ P_1 = 1,\ \text{at } \zeta = 0 \\ F_1' = 0,\ G_1 = 0,\ H_1 = 0,\ P_1 = 0, \quad \text{at } \zeta \to \infty \end{array}\right\} \tag{31}$$

For the second order; $O\left(\varepsilon^2\right)$:

$$F_2''' + F_2'' + F_1 F_1''' - M_1 F_1' + \left(4R - M_2\right) G_1 + G_r H_1 + G_m P_1 = 0 \tag{32}$$

$$G_2'' + G_2' + F_1 G_1' + \left(M_2 - 4R\right) F' - M_1 G_1 = 0 \tag{33}$$

$$H_2'' + P_r H_2' + P_r F_1 H_1' = 0 \tag{34}$$

$$P_2'' - 2S_c P_1 F_1' + S_c P_2' + S_c F_1 P_1' + S_0 S_c H_2'' = 0 \tag{35}$$

$$\left.\begin{array}{l} F_2 = 0,\ F_2' = 1,\ G_2 = 0,\ H_2 = 0,\ P_2 = 0,\ \text{at } \zeta = 0 \\ F_2' = 0,\ G_2 = 0,\ H_2 = 0,\ P_2 = 0, \quad \text{at } \zeta \to \infty \end{array}\right\} \tag{36}$$

For the third order; $O\left(\varepsilon^3\right)$:

$$F_3''' + F_3'' + F_1 F_2'' + F_2 F_1'' - M_1 F_2' + \left(4R - M_2\right) G_2 + G_r H_2 + G_m P_2 = 0 \tag{37}$$

$$G_3'' + G_3' + F_1 G_2' + F_2 G_1' + \left(M_2 - 4R\right) F_2' - M_1 G_2 = 0 \tag{38}$$

$$H_3'' + P_r F_1 H_2' + P_r H_3' + P_r F_2 H_1' + = 0 \tag{39}$$

$$P_3'' - 2S_c P_2 F_1' - 2S_c P_1 F_2' + S_c P_3' + S_c F_1 P_2' + 2S_c F_2 P_1' + S_0 S_c H_3'' = 0 \tag{40}$$

$$\left.\begin{array}{l} F_3 = 0,\ F_3' = 0,\ G_3 = 0,\ H_3 = 0,\ P_3 = 0,\ \text{at } \zeta = 0 \\ F_3' = 0,\ G_3 = 0,\ H_3 = 0,\ P_3 = 0, \quad \text{at } \zeta \to \infty \end{array}\right\} \tag{41}$$

etc.

The solutions of the above equations up to order 3 under the prescribed boundary conditions are obtained in a straightforward manner and are

$$F_1 = 1 - e^{-\zeta} \tag{42}$$

$$G_1 = 0 \tag{43}$$

$$H_1 = e^{-P_r \zeta} \tag{44}$$

$$P_1 = A_1 e^{-P_r \zeta} + A_2 e^{-S_c \zeta} \tag{45}$$

$$F_2 = 0.25 e^{-2\zeta} + A_3 \zeta e^{-\zeta} + A_4 e^{-P_r \zeta} + A_5 e^{-S_c \zeta} + A_6 e^{-\zeta} + A_7 \tag{46}$$

$$G_2 = \left(M_2 - 4R\right)\zeta e^{-\zeta} \tag{47}$$

$$H_2 = A_8 e^{-P_r\zeta} - \zeta P_r e^{-P_r\zeta} - A_8 e^{-\zeta(1+P_r)} \tag{48}$$

$$P_2 = A_9 e^{-\zeta(1+P_r)} + A_{10} e^{-\zeta(1+S_c)} + A_{11} e^{-P_r\zeta} - \zeta A_{12} e^{-S_c\zeta} + \zeta A_{13} e^{-P_r\zeta} + A_{14} e^{-S_c\zeta} \tag{49}$$

$$\begin{aligned} F_3 = &-0.07e^{-3\zeta} - 0.5A_3\zeta e^{-2\zeta} - A_3 e^{-2\zeta} - 0.5A_{15}\zeta^2 e^{-\zeta} + 0.25A_{16} e^{-2\zeta} + A_{17}\zeta e^{-S_c\zeta} + A_{18}\zeta e^{-P_r\zeta} \\ &- A_{19} e^{-\zeta(1+S_c)} - A_{20} e^{-\zeta(1+P_r)} + A_{21} e^{-\zeta} + A_{22} + A_{23} e^{-S_c\zeta} + A_{24} e^{-P_r\zeta} + A_{25}\zeta e^{-\zeta} \end{aligned} \tag{50}$$

$$G_3 = -0.5\left(M_2 - 4R\right)\zeta e^{-2\zeta} + 0.5A_{26}\zeta^2 e^{-\zeta} + A_{27}\zeta e^{-\zeta} + A_{28} e^{-P_r\zeta} + A_{29} e^{-S_c\zeta} + A_{30} e^{-\zeta} \tag{51}$$

$$H_3 = \left(A_{31} + 1\right)\zeta e^{-P_r\zeta} + A_{32} e^{-\zeta(2+P_r)} + A_{36} e^{-\zeta(1+P_r)} - A_{34} e^{-2P_r\zeta} + A_{35}\zeta e^{-\zeta(1+P_r)} + A_{37} e^{-P_r\zeta} + 0.5\zeta^2 e^{-P_r\zeta} \tag{52}$$

$$\begin{aligned} P_3 = &A_{38} e^{-\zeta(2+P_r)} + A_{39} e^{-\zeta(2+S_c)} + A_{40} e^{-2P_r\zeta} + A_{41} e^{-\zeta(P_r+S_c)} + A_{42} e^{-2S_c\zeta} + A_{43}\zeta e^{-\zeta(1+S_c)} \\ &+ A_{44}\zeta e^{-\zeta(1+P_r)} + A_{45}\zeta^2 e^{-S_c\zeta} + A_{46}\zeta^2 e^{-P_r\zeta} + A_{47}\zeta e^{-\zeta(1+P_r)} + A_{48}\zeta e^{-\zeta(1+S_c)} + A_{49} e^{-P_r\zeta} \end{aligned} \tag{53}$$

where the constants $A_i\left(\text{where } i = 1, 2, 3, \ldots.\right)$ are not shown for brevity. The velocity, the temperature and the concentration fields are thus obtained from equations (23) - (26)

$$\frac{u}{U_0} = f'\left(\eta\right) = F_1' + \varepsilon F_2' + \varepsilon^2 F \tag{54}$$

$$\frac{w}{U_0} = g\left(\eta\right) = G_1 + \varepsilon G_2 + \varepsilon^2 G_3 \tag{55}$$

$$\theta\left(\eta\right) = H_1 + \varepsilon H_2 + \varepsilon^2 H_3 \tag{56}$$

$$\phi\left(\eta\right) = P_1 + \varepsilon P_2 + \varepsilon^2 P_3 \tag{57}$$

Thus with the help of equations (42)–(53) the velocity, temperature and concentration distributions can be calculated from equations (54) – (57). The velocity distributions are shown in Figs 2- 15.

Skin-friction coefficient, Nusselt number & Sherwood number

The quantities of chief physical interest are the skin friction coefficients, Nusselt number and Sherwood number.

The equations defining the wall skin friction are $\tau_x = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$

and $\tau_z = \mu\left(\frac{\partial w}{\partial y}\right)_{y=0}$

Thus from equations (54) and (55) we have, $\tau_x \propto f''(0)$ and $\tau_z \propto g'(0)$. The Nusselt number denoted by N_u is proportional to $-\left(\frac{\partial T}{\partial y}\right)_{y=0}$, hence we have from (56), $N_u \propto -\theta'(0)$

The Sherwood number denoted by S_h is proportional to $-\left(\frac{\partial C}{\partial y}\right)_{y=0}$, hence we have from (57), $S_h \propto -\phi'(0)$

The skin friction coefficients, Nusselt number and Sherwood number are respectively obtained from the above equations and these values are sorted in Tables 1-3.

Results and Discussion

For the purpose of discussing the results some numerical calculations are carried out for non-dimensional primary $(f'(\eta))$ and secondary $(g(\eta))$ velocities. The velocity profiles for the x and z components of velocity are shown in figures 2 – 13 for different values of β_e, β_i, S_0, f_w, R, S_c and P_r. The value of M is taken to be large which corresponds to a strong magnetic field. The value of G_r is taken to be large (G_r =10) since the value corresponds to a cooling problem that is encounter in nuclear engineering in connection with the cooling of reactors. Negative values of G_r and G_m,

which indicates the heating of the plate by free convection currents, are also taken into account. For Prandtl number P_r three values 0.71, 1.0 and 7.0 are considered (0.71)

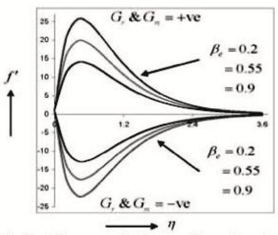

Fig.2 Primary velocity profiles due to cooling and heating plate for $P_r = 0.71$, $S_c = 0.6, f_w = 3.0, R = 0.2, M = 5.0, S_0 = 1.0, \beta_i = 0.1$

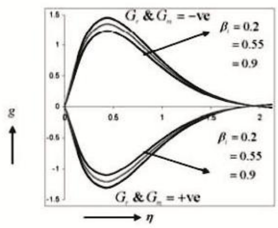

Fig.5 Secondary velocity profiles due to cooling and heating for $P_r = 0.71$, $M = 5$, $S_0 = 1$, $\beta_e = 0.1$, $f_w = 3$, $R = 0.2$, $S_c = 0.6$

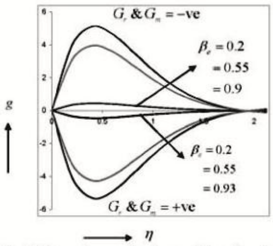

Fig.3 Secondary velocity profiles due to cooling and heating for $P_r = 0.71$, $M = 5$, $S_c = 0.6, S_0 = 1, \beta_i = 0.1, f_w = 3, R = 0.2$

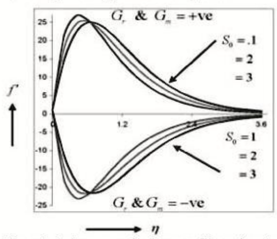

Fig. 6 Primary velocity profiles due to cooling and heating plate for $P_r = 0.71$, $S_c = 0.6$, $M = 5$, $\beta_e = 0.1$, $\beta_i = 0.1$, $f_w = 3, R = 0.2$

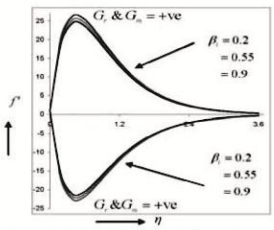

Fig.4 Primary velocity profiles due to cooling and heating plate for $P_r = 0.71$, $S_c = 0.6, f_w = 3.0, R = 0.2, M = 5.0, S_0 = 1.0, \beta_e = 0.1$

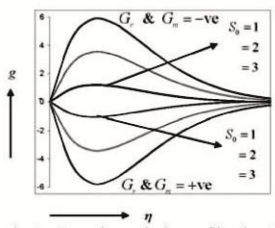

Fig. 7. Secondary velocity profiles due to cooling and heating for $P_r = 0.71$, $M = 5$, $S_c = 0.6$, $\beta_e = 0.1$, $\beta_i = 0.1$, $f_w = 3$, $R = 0.2$

represents air at 20^0C, 1.0, 1.0, 1.0 corresponds to electrolytic solution such as salt water and 7.0 correspond to water) .The value of S_c is taken to be 0.22, 0.6 and 0.75 which corresponds to Hydrogen, water vapor and Oxygen respectively. The values of G_r, G_m, β_e, β_i, So, *R*, *M* and f_w are however chosen arbitrarily with the above mentioned parameters, Figs. 2 and 3 represents the velocity profiles of primary velocity (f') and secondary velocity (g) for different values of β_e. It is found that for cooling plate, the primary velocity (f') and secondary velocity (*g*) decrease with the increase of β_e.

Figures 4 and 5 represents the velocity profiles of primary velocity (f') and secondary velocity (g) for different values of β_i. It is found that for cooling plate (positive values of G_r and G_m), both primary f' & secondary g velocities decrease with the increase of β_i. Fig. 6 represents the velocity profiles of primary velocity (f') for different values of S_0. It is found that for cooling plate (positive values of G_r and G_m) f' increases up to the point $0 < \eta \leq 0.65$ and decreases from this point $0.65 \leq \eta < 3.6$ with the increase of S_0.

Figures 7 represents the velocity profiles of secondary velocity (g) for different values of S_0. It is seen that for cooling plate (positive values of G_r and G_m), *g* decreases with the increase of S_0. Figs. 8 and 9 represent the velocity profiles of primary velocity (f') and secondary velocity (g) for different values of f_w.

Figures 8 and 9 depict that an increase in the suction parameter leads to a uniform decrease both primary and secondary velocities in case of cooling plate. The usual stabilizing effect of the suction parameter (f_w) on the boundary layer growth is also evident from these figures. Figures 10 and 11 represent the velocity profiles of primary velocity (f') and secondary velocity (g) for different values of R. It is found that for cooling plate (positive values of G_r and G_m), f' has a minor decreasing effect while g has a larger decreasing effect with the increase of R. Figures 12 & 13 represent the velocity profiles of primary velocity (f') and secondary velocity (g) for different values of S_c in case of cooling plate. It is found that the primary (f') and secondary velocity (g) decrease with the increase of S_c. It is found that for cooling of the plate (positive values of G_r and G_m), secondary velocity (g) decreases with the increase of S_c but when $S_c =$ 0.75 (corresponding to Oxygen) the value of g is more than the values for $S_c =$ 0.22 and 0.6. Figures 14 and 15 represent the velocity profiles of primary velocity (f') and secondary velocity (g) for different values of P_r. It is found that for cooling plate, f' and g both decreases with the increase of P_r from 0.71 to 7.00. In all the figures mentioned above, compared to the case of cooling of the plate, opposite effects is observed in case of heating of the plate.

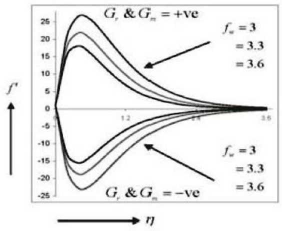

Fig. 8 Primary velocity profiles due to cooling and heating plate for $P_r = 0.71$, $\beta_e = 0.1, S_0 = 1, S_c = 0.6, M = 5, \beta_i = 0.1, R = 0.2$

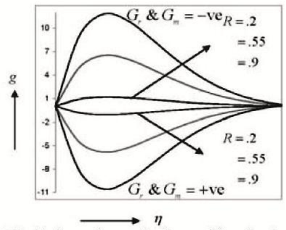

Fig.11 Secondary velocity profiles due to cooling and heating of the plate for $\beta_e = 0.1$, $\beta_i = 0.1, S_0 = 1, P_r = 0.71, M = 5, S_c = 0.6, f_w = 3$

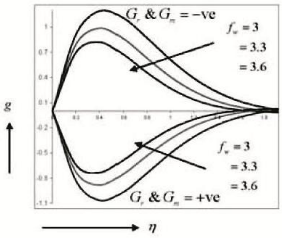

Fig. 9 Secondary velocity profiles due to cooling and heating of the plate for $\beta_e = 0.1$, $\beta_i = 0.1, S_0 = 1, P_r = 0.71, M = 5, S_c = 0.6, R = 0.2$

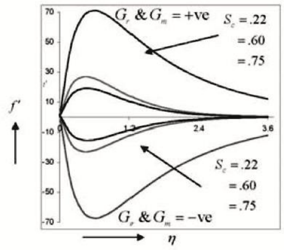

Fig. 12 Primary velocity profiles due to cooling and heating plate for $P_r = 0.71$, $M = 5, \beta_e = 0.1, \beta_i = 0.1, f_w = 3, R = 0.2$ $S_0 = 1$,

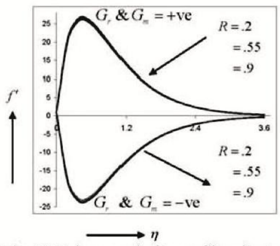

Fig. 10 Primary velocity profiles due to cooling and heating plate for $P_r = 0.71$, $S_c = 0.6, M = 5, \beta_e = 0.1, \beta_i = 0.1, S_0 = 1, f_w = 3$

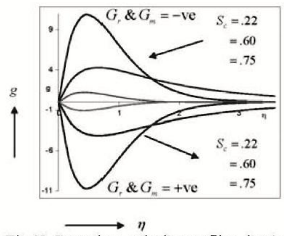

Fig.13 Secondary velocity profiles due to cooling and heating of the plate for $\beta_e = 0.1$, $\beta_i = 0.1, P_r = 0.71, M = 5, S_0 = 1, f_w = 3, R = 0.2$

Finally, in table 1-3 numerical values of the Skin frictions, Nusselt number and Sherwood number are tabulated. The effect of various parameters on the Skin frictions, Nusselt number and Sherwood number as observed from these tables are found to agree with the effects on the velocity profiles.

Table 1: *Numerical values proportional to* τ_z, N_u, *and* S_h *for* $\beta_e = 0.6$, $\beta_i = 0.1$, $P_r = 0.71$, $S_c = 0.6$, $M = 5$ *and* $S_0 = 1$

G_r	G_m	R	f_w	τ_x	τ_z	N_u	S_h
10	4	0.2	3	-371.746	0.694725	0.708865	0.55938
10	4	0.3	3	-371.655	0.467339	0.708865	0.55938
10	4	0.2	3.3	-306.832	0.60782	0.708968	0.524346
-10	-4	0.2	3	371.7283	-0.85017	0.708865	0.55938
-10	-4	0.3	3	371.8191	-0.57191	0.708865	0.55938
-10	-4	0.2	3.3	306.2459	-0.66896	0.708968	0.524346

Table 2: *Numerical values proportional to* τ_x, τ_z, N_u *and* S_h *for* $R = 0.2$, f_w =3, $P_r = 0.71$, $S_c = 0.6$, $M = 5$ *and* $S_0 = 1$

G_r	G_m	β_e	β_i	τ_x	τ_z
10	4	0.3	0.1	-454.005	0.235659
10	4	0.6	0.1	-371.746	0.694725
10	4	0.6	0.3	-352.875	0.539235
-10	-4	0.3	0.1	455.7263	-0.40061
-10	-4	0.6	0.1	371.7283	-0.85017
-10	-4	0.6	0.3	352.7218	-0.61357

Table 3: *Numeroical values proportional to* τ_x, τ_z, N_u *and* S_h *for* β_e = 0.6, $\beta_i = 0.1$, $R = 0.2$, f_w =3, $M = 5$ *and* $S_0 = 1$

G_r	G_m	P_r	S_c	τ_x	τ_z	N_u	S_h
10	4	0.71	0.22	-254.77	25.17645	0.708865	0.230152
10	4	0.71	0.60	-371.746	0.694725	0.708865	0.55938
10	4	7	0.60	-138.719	2.438216	-21.2917	-3.20313
-10	-4	0.71	0.22	254.7525	-25.3319	0.708865	0.230152
-10	-4	0.71	0.60	371.7283	-0.85017	0.708865	0.55938
-10	-4	7	0.60	138.7017	-2.59367	-21.2917	-3.20313

Conclusions

1. For cooling plate (positive values of G_r and G_m), f' increases up to the point $0 < \eta \leq 0.65$ and decreases from this point $0.65 \leq \eta < 3.6$ with the increase of S_0 while secondary velocity g, decreases with the increase of the Soret number S_0.
2. An increase in the suction parameter leads to a uniform decrease both primary and secondary velocities in case of cooling plate. The usual stabilizing effect of the suction parameter (f_w) on the boundary layer growth is also evident from these figures.
3. For cooling of the plate (positive values of G_r and G_m), secondary velocity (g) decreases with the increase of S_c but when $S_c =$ 0.75 (corresponding to Oxygen) the value of g is more than the values for $S_c =$ 0.22 and 0.6
4. For cooling plate, f' and g both decreases with the increase of P_r from 0.71 to 7.00

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